WATERSHED SEGMENTATION OF AN IMAGE ENHANCED BY TEAGER ENERGY DRIVEN DIFFUSION

D. De Vleeschauwer1, F. Alaya Cheikh2, R. Hamila2, M. Gabbouj2

1University of Ghent, Belgium
2Tampere University of Technology, Finland

ABSTRACT

In this paper we use the Teager energy driven non-linear diffusion process to reduce the influence of noise in the watershed segmentation of an image. We show that the performance of this non-linear diffusion process is better than the performance of the traditional gradient driven non-linear diffusion process.

1. INTRODUCTION

The scale space of an image, introduced by Witkin (1) and Koenderink (2), consists of versions of that image at different scales. The image \( I(x,y,t) \) at scale \( t \) is the original image \( I(x,y,0) \) convolved with a Gaussian pulse with width proportional to \( \sqrt{t} \). The set of images in the scale space obeys the linear diffusion equation (see eq. (1) with \( c(x,y,t)=1 \). Consequently, the scale space is efficiently built with a diffusion process: i.e. the image at scale \( t \) is calculated by local averaging the image at scale \( t_0 \) with a small, place-invariant, linear filter.

As the scale \( t \) increases, the noise of the original image is reduced, but the edges in the image get blurred too. To preserve the edge sharpness, while reducing the noise as the scale \( t \) increases, Perona and Malik (3) introduced a scale space based on non-linear diffusion (see eq. (1)). In their non-linear diffusion process local averaging is inhibited in regions where edges are expected and it is allowed to go on as before in other regions. This diffusion behavior is achieved, if the local diffusion velocity \( c(x,y,t) \) depends on the amplitude of the gradient. If the gradient is large, the presence of an edge is expected and diffusion is inhibited by choosing the local diffusion velocity \( c(x,y,t) \) low. If the gradient is small the local diffusion velocity \( c(x,y,t) \) can take its maximal value. Saint-Marc et al (4) describe a similar adaptive smoothing technique.

The amplitude of the gradient is crude way to detect the presence of an edge. In this paper we use the two-dimensional Teager energy, introduced by Alaya Cheikh et al (5), instead of the amplitude of the gradient. This two-dimensional Teager energy, an extension of the one-dimensional Teager energy introduced by Kaiser (6), discriminates better between noise peaks and true edges. This makes the performance of Teager energy driven non-linear diffusion introduced in this paper, better than the performance of traditional gradient driven non-linear diffusion.

In this paper the performance of the non-linear diffusion process is assessed by following the segments found by watershed segmentation as the scale \( t \) increases. Watershed segmentation (see e.g. Vincent and Soille (7)) of the image at increasing scales \( t \) is ideally suited to illustrate the performance of the non-linear diffusion process. In the original image, i.e. the image at scale \( t=0 \), noise tends to shatter the segments corresponding to objects in a lot of small pieces, even when there is only a small amount of noise. The non-linear diffusion process is able to gradually remove this noise. At larger scales \( t \) the segments estimated with the watershed algorithm correspond better to homogeneous parts of the objects. The lower the scale \( t \) at which the segments found by the watershed segmentation are good enough, the better the performance of the non-linear diffusion process.

In the next section, the segmentation of an image cleaned by a Teager energy driven non-linear diffusion process is described. Then, in section 3, the performance of the non-linear diffusion process is assessed. Finally, in the last section, some conclusions are drawn.

2. SEGMENTATION OF THE DIFFUSED IMAGE

Teager energy driven diffusion

The images \( I(x,y,t) \) in the scale space, with \( I(x,y,0) \) the original image and \( t \) the scale parameter, obey the diffusion equation:

\[
\nabla \cdot [c(x,y,t) \nabla I(x,y,t)] = \frac{\partial I(x,y,t)}{\partial t}.
\]

In the linear scale space of Witkin (1) and Koenderink (2) the diffusion velocity is constant: \( c(x,y,t)=1 \). In the non-linear scale space the diffusion velocity depends on a local activity that indicates the presence of an edge. Perona and Malik (3) used the (squared) amplitude of the gradient as activity image.
\[ c(x, y, t) = g \left( \left\| \nabla I(x, y, t) \right\|^2 \right) \]

The function \( g(.) \) is a soft threshold function:

\[ g(u) = \frac{c_{\text{max}}}{1 + \left( \frac{u}{T_D} \right)^\alpha} \]

Under the threshold \( T_D \), the function \( g(.) \) takes approximately the value \( c_{\text{max}} \); above the threshold \( T_D \) the function \( g(.) \) tends to 0. The parameter \( \alpha \) determines the steepness of the function \( g(.) \) at the threshold. The larger this parameter \( \alpha \) is, the steeper the function \( g(.) \) is, and the more crisp the threshold is. In Perona and Malik (3) the steepness parameter \( \alpha \) is equal to 1.

In this paper we study two extensions of the non-linear diffusion process of Perona and Malik (3). As a first extension, we use the Teager energy, instead of the squared gradient, as activity image:

\[ c(x, y, t) = g \left( E(x, y, t) \right) \]

The Teager energy reflects better the local activity than the amplitude of the gradient. The (discrete) Teager energy \( E_{m,n} \), originally defined for one-dimensional signals, was extended to two dimensions by Alaya Cheikh et al (5):

\[ E_{m,n} = \max \left\{ E_{m,n}^r, E_{m,n}^d \right\} \]

with:

\[ E_{m,n}^r = 2 \left( I_{m,n} - \mu_{m,n} \right)^2 - \left( I_{m-1,n} - \mu_{m,n} \right) \left( I_{m+1,n} - \mu_{m,n} \right) \]

\[ E_{m,n}^d = 2 \left( I_{m,n} - \mu_{m,n} \right)^2 - \left( I_{m,n-1} - \mu_{m,n} \right) \left( I_{m,n+1} - \mu_{m,n} \right) \]

where \( I_{m,n} \) is the image sample at location \((m,n)\) and \( \mu_{m,n} \) is the local mean. They used this two-dimensional Teager energy to reduce noise in an image with an adaptive filter: The Teager energy determined the local filter coefficients.

As a second extension, the optimal value for the steepness parameter \( \alpha \) is searched for. It is expected that as in Alaya Cheikh et al (5), values smaller than 1 will be found to be optimal.

**Watershed segmentation**

To illustrate the noise removal ability of the non-linear diffusion process, the image is segmented at different scales \( t \) with a watershed algorithm. We use a similar watershed algorithm as the one developed by Vincent and Soille (7). First, the activity image at scale \( t \) is calculated. In the case of the gradient driven diffusion, this activity image is the squared amplitude of the gradient. In the case of the Teager energy driven diffusion this activity image is the Teager energy. The value of the pixel in the activity image is high if the pixel is an edge pixel and is low if the pixel is an interior pixel. This activity image gets less noisy as the scale \( t \) increases. The activity image forms a three-dimensional surface, with "mountains" where edges are expected and "valleys" in the interior pixels, upon which the watershed process is performed.

The watershed process itself consists of two steps. In the first step all neighboring pixels whose activity value lies below a threshold \( T_w \) are grouped together. These valley regions form the attraction basins (see Vincent and Soille (7)). Then, in a second step the pixels that are not yet classified are considered. Such a pixel is classified to the same region as the one of its eight neighbors whose activity value is most below the value of the pixel under consideration. That pixel lies in the direction a droplet of water would flow if it would hit the activity surface in the considered pixel. In this second step the attraction basins of the first step grow and occasionally new attraction basins are formed. Eventually each pixel gets classified, and all segments together cover the complete image.

Watershed algorithms are extremely sensitive to noise. Regions corresponding to real objects are broken up in lots of different pieces, even when only a small amount of noise is present. Therefore, noise has to be removed. The aim of this paper is to show that the Teager energy driven non-linear diffusion removes the noise better than the gradient driven non-linear diffusion.

**The causality property**

Perona and Malik (3) argued that a (non-linear) diffusion process ought to have the causality property. This means that as the scale \( t \) increases no "new" features (of a certain type) must be created. Witkin (1) proved that linear diffusion exhibits this property with as features the zero-crossings of the Laplacian: He showed that as linear diffusion progresses such zero-crossings can only merge; a zero-crossing at a larger scale \( t_{k-1} \) can be traced back to a zero-crossing at a lower scale \( t_k \). Perona and Malik (3) proved that the
gradient driven non-linear diffusion process possesses
the causality property with as features the extrema in
the image, if the velocity function \( g(\cdot) \) of eq. (2) is well
chosen. This proof is no longer valid for other types of
features, particularly for the features used in this paper,
i.e. the segments found by the watershed algorithm.
There is also no theoretical proof for the causality
property of Teager energy driven non-linear diffusion
for any kind of features.

If non-linear diffusion would possess the causality
property, it would be guaranteed that segments could
only merge as scale \( t \) increases. Since there are no
theoretical proofs, we verify this experimentally in the
following section.

3. PERFORMANCE EVALUATION

The test image “block”

Figure 1 shows the original test image “block”. The
image is simple enough so that it can be segmented by
hand in 6 segments (the background and 5 facets).
Closely examining the image reveals that this image,
especially the background, contains some noise,
making the automated segmentation far from trivial.

To better illustrate the performance of the two non-
linear driven diffusion processes, this original image
was further disturbed with noise. Two kinds of noise
were considered: additive Gaussian noise and impulsive
noise (i.e. salt and pepper noise). The standard
deviation of the additive Gaussian noise was 5% of the
image range (i.e. 255*0.05). For the impulsive noise
1% of the pixels were randomly selected and
substituted with a random value.

Figure 1: The image “block”.

Comparison of Teager energy driven and gradient
driven non-linear diffusion

Figure 2 shows how the number of segments evolves as
the non-linear diffusion of eq. (1) progresses. The
performance on three images is shown: on the original
image “block” and on the image “block” disturbed by
additive Gaussian noise and the image “block”
disturbed by impulsive noise. The two non-linear
diffusion processes are compared: the one where the
diffusion velocity is determined by the gradient (see eq.
(2)) and the one where this diffusion velocity is
determined by the Teager energy (see eq. (4)). In these
experiments the steepness parameter \( \alpha \) is 1, the
threshold \( T_g \) is chosen about equal to the average of the
activity image, and the threshold \( T_e = 0.025T_g \).

Figure 2: Number of segments remaining
after non-linear diffusion up to scale \( t \) in the
original image and the noisy images for the
Teager energy driven diffusion and the
gradient driven diffusion.

Without diffusion the 6 segments that are obvious, if
the image is segmented by hand, are scattered in lots of
small pieces. Especially the background suffers from
this burden. As the diffusion progresses these segments
are grouped, and eventually, if the non-linear diffusion
process performs well, the 6 segments are found. If the
non-linear diffusion goes on for too long, the edges get
blurred and this causes the segment boundaries to be
dislocated. Therefore, for this image only diffusion up
to scale \( t = 7.0 \) was considered.

Notice that the curves are all monotonously decreasing.
Less and less segments are found as the non-linear
diffusion progresses. This illustrates that, at least for
this example, the non-linear diffusion processes
exhibits the causality property for the segments found
by the watershed algorithm.

Figure 2 also illustrates that the segmentation of the
noisy images is more difficult than the segmentation of
the original image: i.e. non-linear diffusion up to a
larger scale is required for noisy images. Additive
Gaussian noise is removed by both non-linear diffusion
processes, but the Teager energy driven non-linear diffusion removes this type of noise marginally better than the gradient driven non-linear diffusion. Impulsive noise is very disturbing. The gradient driven non-linear diffusion is not able to remove impulsive noise. The Teager energy driven non-linear diffusion has also in this case a reasonable performance.

The values of the thresholds $T_x$ and $T_{\omega}$

To set the threshold $T_x$ for the diffusion and the threshold $T_{\omega}$ for the watershed segmentation, the average value of the activity image (the squared amplitude of the gradient or the Teager energy) is calculated.

If the threshold $T_x$ in the non-linear diffusion process is set too high, no activity value reaches above this threshold. In that case the non-linear diffusion process reduces to a linear diffusion process, because $c(x,y,t)=c_{max}$. Figure 3 illustrates the effect of this threshold $T_x$ on the segments of the image “block”. Especially the (sharp) corners are distorted when the threshold $T_x$ is set too high. If the threshold $T_x$ is set too low, all diffusion velocities are small, i.e. $c(x,y,t)=0$, and the image does not change as the scale $t$ increases. Therefore, the threshold $T_x$ is always chosen about equal to the average of the activity image.

If the threshold $T_{\omega}$ in the watershed segmentation, is set too high, all activity values lie below this threshold and the whole image is just one attraction basin. When this threshold is set too low, noise has too much influence. Experimentally it was determined that $T_{\omega} \in [0.01 T_x, 0.17 T_x]$ are good values.

The optimal value of the steepness parameter $\alpha$

Figure 4 shows how the number of segments evolves as the Teager energy driven non-linear diffusion progresses on the original image “block” for various values of the steepness parameter $\alpha$.

Remark that the curves are all monotonously decreasing. If the steepness parameter $\alpha$ (>2) is chosen too large, there tend to be some causality problems. This is reason enough to choose the steepness parameter small enough.

Figure 4: Number of segments remaining after Teager energy driven non-linear diffusion up to scale $t$ in the image “block” for various values of the parameter $\alpha$.

Figure 4 shows that the optimal value of the steepness parameter $\alpha$ lies around 0.7 or 0.8. At the start ($t<3.0$) of the non-linear diffusion process a value of 0.5 is even better, but at higher scales ($t>4.0$) a value just below 1 is best. The value 0.8 is the best compromise. This corresponds with the values found by Alaya Cheikh et al (5).

Fractional values for the steepness parameter $\alpha$ consume more CPU time (see eq.(3)). If CPU time is of no concern, the optimal value of 0.8 is chosen; Otherwise the value 1 is chosen for the steepness parameter $\alpha$.

Performance on a complex scene

Figure 5 shows the image “peppers”. It is used to illustrate the segmentation of an image cleaned by Teager energy driven non-linear diffusion on a more complex scene.

Figure 6 shows the image “peppers” segmented with the watershed algorithm after Teager energy non-linear diffusion up to scale $t=8$. The parameter $\alpha$ is 0.8, the threshold $T_x$ is set to about half the average Teager energy, and the threshold $T_{\omega}=0.02 T_x$.

4. CONCLUSION

In this paper the Teager energy driven non-linear diffusion process was introduced.
Comparing its performance to the traditional gradient driven non-linear diffusion process, it was concluded that the Teager energy driven non-linear diffusion was much better at removing impulsive noise and slightly better at removing Gaussian noise.

Furthermore, the optimal steepness parameter $\alpha$ for the threshold function was determined. It was proven that a value slightly smaller than 1 was optimal.

The performance of the method was also illustrated on a complex scene.

Figure 5: The image “peppers”.

Figure 6: The segments of the image “peppers” after Teager energy driven non-linear diffusion up to scale $r=8$.

REFERENCES


