ABSTRACT
Color image interpolation using Directional-Vector Rational Filters (DVRF) is investigated in this paper. We study this problem (or the upampling problem) for two decimation schemes that are most commonly used in practice: (1) rectangular decimation (every-other-row and every-other-column sub-sampling) and (2) Quincunx decimation. For each decimation lattice, we present an adaptive re-sampling algorithm using a new non-linear interpolator, which has desirable properties, such as the preservation of edges and image details, and the preservation of inter-channel correlations.

Extensive simulations show that the proposed algorithms outperform linear and non-linear techniques, e.g., vector FIR median hybrid filters (VFHM). Image interpolated using DVRF are free from blockiness and jaggedness.

Keywords: Rational Filters, Nonlinear Interpolation, Vector Rational Filters, Adaptive processing, Directional Filters.

1. INTRODUCTION

The process of decimation or down-sampling, is an effective way and often used to reduce image sizes; thus, reducing the amount of information transmitted through the communication channels and the local storage requirements, while trying to preserve as much as possible the image quality. Conversely, the reverse procedure of this, referred to as interpolation or up-sampling, is useful in restoring the original high resolution image from its decimated version or for resizing or zooming a digital image. Decimation and interpolation are used for several purposes in many practical applications, such as progressive image transmission systems, image zooming, photographic enlarging, image reconstruction, optical scanners, high resolution printer, and in multi-media applications which require browsing or retrieval of images from the internet or image and video databases. These problems are further aggravated in the case of color images which usually require larger storage capacity and processing time. According to Fig.1, design of robust and good interpolators is very crucial in achieving a reconstructed image with high quality.

A number of conventional interpolation techniques have been proposed in the literature to increase the spatial resolution of an image [4], [5], [6]. These techniques degrade quality of the magnified image due to various artifacts (which are quite sensitive to human eyes), such as, blocking artifact and excessive smoothing. Those degradations become worse as the magnification ratio increases. And there also exists tradeoff between reducing blocking artifact and excessive smoothness [3]. Moreover, the conventional techniques are well established methods for univariate two-dimensional signals, such as grey level images. An extension of these techniques to multivariate data, such as color images is not straightforward. Processing each color component separately will fail to take into account the inherent correlation which exists between the different channels (i.e., colors).

Adaptive methods aim to avoid these problems by analyzing the local structure of the source image and using different interpolation functions with different areas of support.

In the following, we present an adaptive algorithm for color image interpolation using a new class of vector rational filters recently proposed in [1]. It is well known, in fact, that a rational function (the ratio of two polynomials) has several properties [7], [9] (i.e., it is a universal approximator and a good extrapolator, can be trained using an adaptive algorithm, and requires lower degree terms than Volterra expansions) which can make it very effective in many signal processing tasks. In this study, VRF are weighted by vector directional...
filters (VDF) \cite{10} and form the directional-vector rational filters DVRF.

Moreover, the algorithm using this new approach yields better interpolated images than those obtained with a number of linear and nonlinear techniques. Two different re-sampling (i.e. down-sampling/up-sampling) schemes are investigated, the rectangular and Quintcunx lattices.

An outline of this paper is as follows. Section 2 describes the directional-vector rational functions. The adaptive up-sampling algorithm is given in section 3. Section 4 presents our examples and discuss the improvement given by the proposed algorithm using the DVRF; while, section 5 concludes the paper.

2. DIRECTIONAL-VECTOR RATIONAL FUNCTIONS

A straightforward application of a rational functions to multichannel image processing are based on processing the image channels separately. However, these fail to utilize the inherent correlation that is usually present in multichannel images. Consequently, vector processing of multichannel images is desirable \cite{8}, and rational functions are extended in a way that allows them to process color images. Each vector may be described as 3-component vector in the color space, e.g. (R, G, B) space.

The first interpolative scheme is based on border-preserving. With reference to Fig.3, our nonlinear interpolator operates on four samples of decimated data, \( a, b, c \) and \( d \) to reconstruct the missing sample \( z \) in the central position. In order to weight the contributions to \( z \) of its four neighborhood samples, a rational function is used based on vector directions. Also the directional form of the weights is performed to maintain the edge in an arbitrary orientation and consequently to sustain the sharpness of the interpolated image. The interpolator output is as follows:

\[
y \left( u, v \right) = \frac{\sum_{u \neq v, u, v \in \{a, b, c, d\}} w_{uv} (u + v)}{2 \sum_{u \neq v, u, v \in \{a, b, c, d\}} w_{uv}} \tag{1}
\]

where

\[
w_{uv} = \frac{1}{12 + kA^2(u, v)} \tag{2}
\]

The parameter \( k \) is some positive constant and is used to control the amount of the nonlinear effect, and \( u, v \in \{a, b, c, d\} \). \( A(u, v) \) denotes the angle between the two directions of the vectors \( u \) and \( v \) (expressed in radians) as it can be seen in Fig. 5. In general:

\[
0 \leq A(u, v) \leq \frac{\pi}{2} \tag{3}
\]

Whereas for the case of color images:

\[
0 \leq A(u, v) \leq \frac{3\pi}{2} \tag{4}
\]

Another interpolation scheme can be implemented using two independent masks given by Fig.2-c, (The missing samples \( x \) and \( y \) can be interpolated by using respectively, row mask and column mask). The interpolated sample \( x \) is computed as:

\[
x = \alpha v_2 + (1 - \alpha) v_3 \tag{5}
\]

where

\[
\alpha = \frac{1 + kA^2(v_2, v_4)}{2 + k(A^2(v_1, v_3) + A^2(v_2, v_4))} \tag{6}
\]

Sample value \( y \) is computed using a similar mask but column wise. To compute the missing samples \( x \) in Fig.3-a, using the already interpolated samples, we apply Eq.(5) to the horizontal and vertical masks centered in \( z \), and the mean value is assigned to \( z \).

In the following, we describe the interpolation algorithm using the combination of the two schemes described by Eq.(1) and Eq.(5).

3. ADAPTIVE ALGORITHM

The main purpose of the proposed interpolator is to maintain the fidelity of the interpolated image by preserving the edges of the original image.

3.1. Rectangular decimation

To achieve this goal, the two interpolator schemes given by Eq.(1) and Eq.(5) can be used adaptively together according to Fig.3-a, as follows:

Step 1: a: Using Eq.(5), compute temporarily the \( x \) samples (row mask);
   b: Using Eq.(4), compute temporarily the \( y \) samples (column mask);

Step 2. To compute the unknown \( x \) samples, we apply the two masks according to Fig.2-a) and using Eq.(1). The mean value of the two masks is assigned to \( x \).

Step 3. Recompute the \( x \) and \( y \) samples using the original pixels and the \( x \) pixels computed in step 2. We apply only the second mask in Fig.2-b) and using Eq.(1) to compute the new values.
3.2. Quincunx decimation

The corresponding algorithm interpolates color images downsampled using the line quincunx decimation scheme, see Fig. 3-b. The interpolation is done in a single step, using a 3x3 window. Given the four input vectors a, b, c and d shown in Fig. 2-b we compute the central pixel according to Eq. (1), with the weights being functions of $L_2$ norm given by Eq. (2).

4. EXPERIMENTAL RESULTS

To assess the performance of our interpolators, two color images, (480x512 Lena image and 512x512 Peppers) are decimated in two different ways: (1) Rectangular decimation with a factor of 1/16, see Fig.3-a, (factor 1/4 is used here for illustration) and (2) Quincunx decimation with a factor 1/2, see Fig.3-b. Only the black points are retained from the decimation process in each of the two decimation methods.

The full size images is then reconstructed using the directional-vector rational interpolators (DVRF) and the $MAE$ and $MSE$ criteria are used to compare quantitatively the performance of our adaptive nonlinear interpolator scheme with those of linear techniques such as the bilinear method (BL) and those of the class of vector FIR median hybrid filters (VFMH) proposed in [2], the cubic convolution method (CC) and the marginal rational filter (MRF).

The tabulated results (see table 1) indicate that most of the nonlinear interpolation methods outperform their linear counterpart. A boldfaced value in a table entry indicates the best filter performance according to the objectively measured MAE and MSE. As can be verified, DVRF result is better, or at least equal performance, in all cases.

Even though these measures are not the best in estimating how good an interpolation scheme is, they can give an idea about the relative performance of the filters. The interpolated images are presented for visual comparison since its in many cases the best qualitative measure of performance for images processing algorithms.

The DVRF performs better than the MRF, this could be explained by the interchannel correlation which is not taken into consideration by the scalar filter.

Figs.4-a), 4-b), 4-c) and 4-d) illustrate the output of the bilinear filter, the vector FIR median hybrid filter, the separable vector rational filter, and the proposed directional-vector rational filter respectively for the rectangular decimation case for Lena image. The aliasing effects are more visible in the VFMH case, while by our interpolator, the processed images exhibit sharp non-jagged edges. The computational complexity of the DVRF is less than the one of VFMH method (small window mask and few operations for each computed sample) and is therefore quite feasible for hardware implementation.

5. CONCLUSIONS

Vector rational filters are weighted by angle differences to process multichannel signals in this paper. The novel directional-vector rational filters are applied to color image interpolation and are shown to preserve edge information and image details, and enable to exploit the existing correlation between the RGB color space. Simulation results show that the proposed interpolators outperform the linear techniques as well as the class of VFMH filters. Some processed images are presented in Figs.4-a), 4-b), 4-c) and 4-d) for qualitative comparison.

Acknowledgment: This work has been supported by the European ESPRIT Project LTR 20229-Noblesse.

6. REFERENCES


Figure 1: Decimation/Interpolation based compression system.

Figure 2: Interpolators: (a) and (b) Bidirectional operator, (c) Unidirectional operator.

Figure 3: Decimation Schemes: (a) Rectangular decimation, (b) Quincunx decimation. Samples in the white boxes denote available data.

Figure 5: Vectorial representation of the color image pixels in the RGB color space.
Table 1: Quantitative measures of the performance of the different interpolators.

<table>
<thead>
<tr>
<th>Method</th>
<th>Rectangular Decimation (1/16)</th>
<th>Quincunx Decimation (1/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lena</td>
<td>Peppers</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>MSE</td>
</tr>
<tr>
<td># CC</td>
<td>21.72</td>
<td>423.60</td>
</tr>
<tr>
<td>BL</td>
<td>19.16</td>
<td>363.50</td>
</tr>
<tr>
<td>VFMH</td>
<td>18.58</td>
<td>346.90</td>
</tr>
<tr>
<td>MRF</td>
<td>18.17</td>
<td>340.06</td>
</tr>
<tr>
<td>DVRF</td>
<td>16.09</td>
<td>329.32</td>
</tr>
</tbody>
</table>

Figure 4. Part of the interpolated results from the (1/16) rectangular decimated images: a) using the Bilinear filter; b) using the vector FIR median hybrid filter; c) the marginal rational filter; d) the Directional-Vector Rational Filter.